Contents

1 Lecture 1

- Agenda
  - What is an algorithm?
  - What is not an algorithm?
  - How is it different than a computer program?
- Foundations
  - Sorting a collection of numbers
    - Permutation Sort
    - Depending on language, the time in seconds changes
      - C, C++, Python, Rust, Lisp
    - Need for the sequence of instructions which is independent of the hardware, programming language,
  - Algorithm
    - Etymology
      - Al Khwarizmi, origins
    - Properties of Algorithms
      - Finiteness: Termination guarantee
      - Definiteness: Precise enough to be communicated to a computer and a fellow human
      - Input: Zero or more inputs
      - Output: Expected output
    - Does a prompt to ChatGPT qualify as an algorithm?
    - Erik Demaine's analogy between the program pyramid and the algorithm pyramid

Program	Algorithm
Programming language	Pseudocode / Structured English
Computer	Model of Computation

    - What is a model of computation?
        - Random Access Machine (RAM) model
        - Primitive (constant time) operations, add, subtract, multiply, integer and floating-point operations.
        - Can there be a constant time sorting operation in-built in the hardware?
        - Other models of computation
        - Parallel RAMs, Quantum Computers, Turing Machines
    - What is a computational problem?
        - A relation on the inputs and outputs
        - [Sorting] A sequence of numbers as the input and its sorted sequence as the output
        - [Shortest Path] A graph with a source and target vertices as input and the shortest path between them as the output
        - [Finding Minimum] A set of numbers as input and the minimum number in the set as the output
        - [Searching] A set of elements and a query element as the input and a yes or no answer as an output

2 Insertion Sort

```
INSERTION-SORT(A, n)
1  for i = 2 to n
2      key = A[i]
3      // Insert A[i] into the sorted subarray A[1 : i − 1].
4      j = i − 1
5      while j > 0 and A[j] > key
6          A[j + 1] = A[j]
7          j = j − 1
8      A[j + 1] = key
```

- A specific algorithm: Insertion Sort
    - Proof of correctness
        - Loop Invariant (Proof by Induction)

        - Initialization (Base step)
        - Maintainence (Inductive hypothesis)
        - Termination (Desired statement)
- Time complexity
    * Count the steps