Data Structures and Algorithms

Lecture 06

Aniket Basu Roy

BITS Pilani Goa Campus

2026-01-23 Fri

Agenda

Recurrence Relations

What to do when we guess it wrong?

- Examples

The Master Theorem

- Examples

Example 1

$$T(n) = 8T(n/2) + \Theta(n^2)$$

Example 1

$$T(n) = 8T(n/2) + \Theta(n^2)$$

Upper Bound

$$T(n) \leq 8T(n/2) + cn^2$$

for some $c > 0$

Example 1

$$T(n) = 8T(n/2) + \Theta(n^2)$$

Upper Bound

$$T(n) \le 8T(n/2) + cn^2$$

for some $c > 0$

- Guess: $\exists d > 0,\ T(n) \le dn^3$

Example 1

$$T(n) = 8T(n/2) + \Theta(n^2)$$

Upper Bound

$$T(n) \le 8T(n/2) + cn^2$$

for some $c > 0$

- Guess: $\exists d > 0,\ T(n) \le dn^3$

WRONG!

Example 1

$$T(n) = 8T(n/2) + \Theta(n^2)$$

Upper Bound

$$T(n) \leq 8T(n/2) + cn^2$$

for some $c > 0$

- Guess: $\exists d, d' > 0,\ T(n) \leq dn^3 - d'n^2$

Example 2

$$T(n) = T(n/3) + T(2n/3) + \Theta(n)$$

The Master Theorem

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1,\ b > 1,\ f(n) \geq 0$

Case 1

If $\exists\varepsilon > 0,\ f(n) = O(n^{\log_b a - \varepsilon})$, then $T(n) = \Theta(n^{\log_b a})$.

Case 2

If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$.

Case 3

If $\exists\varepsilon > 0, f(n) = \Omega(n^{\log_b a + \varepsilon})$ and
$\exists c < 1, n_0 > 0, \forall n \geq n_0,\ \ af(n/b) \leq cf(n)$, then
$T(n) = \Theta(f(n))$.

The Master Theorem

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1,\ b > 1,\ f(n) \geq 0$

Case 1

If $\exists \varepsilon > 0,\ f(n) = O(n^{\log_b a - \varepsilon})$, then $T(n) = \Theta(n^{\log_b a})$.

The Master Theorem

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1,\ b > 1,\ f(n) \geq 0$

Case 2

If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$.

The Master Theorem

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1,\ b > 1,\ f(n) \geq 0$

Case 3

If $\exists \varepsilon > 0, f(n) = \Omega(n^{\log_b a + \varepsilon})$ and
$\exists c < 1, n_0 > 0, \forall n \geq n_0,\ \ af(n/b) \leq cf(n)$, then
$T(n) = \Theta(f(n))$.