Data Structures and Algorithms

Lecture 18

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Agenda

- Data Structures
- Hash Tables
- Chaining
- Hash Functions

The Dictionary Problem

- Given: A dynamic set
- We want to support:
  - INSERT
  - DELETE
  - SEARCH

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- We want to support:
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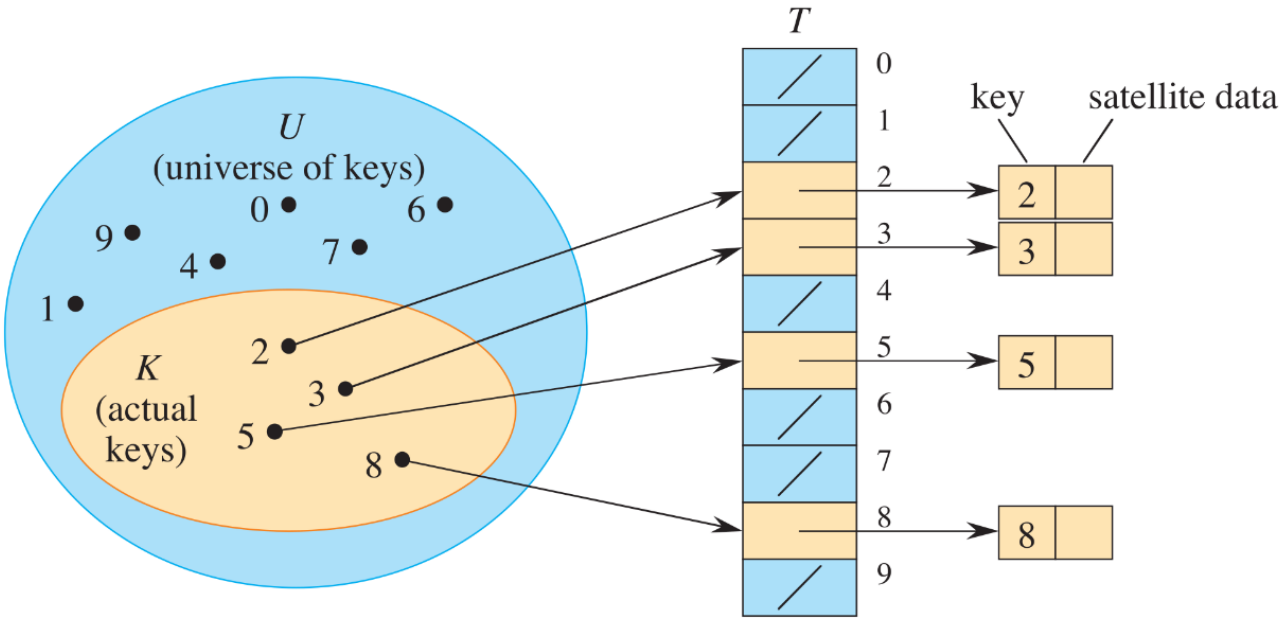
  - SEARCH
- Keys from a universe $U = \{0, 1, \ldots, |U| - 1\}$.
- Goal: $O(1)$ expected time per operation.

The Dictionary Problem



Direct-Address Tables

Allocate array $T[0 : |U| - 1]$. Slot $T[k]$ holds element with key k.

```
DIRECT-ADDRESS-SEARCH(T, k)    return T[k]
DIRECT-ADDRESS-INSERT(T, x)    T[x.key] = x
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Problem: If $|U|$ is large, T requires $\Theta(|U|)$ space — infeasible.

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- Element with key k goes in slot h(k).
- Table size $m \ll |U|$ $\Rightarrow$ space $\Theta(m)$.
- Problem: Two distinct keys may map to the same slot, aka collision.

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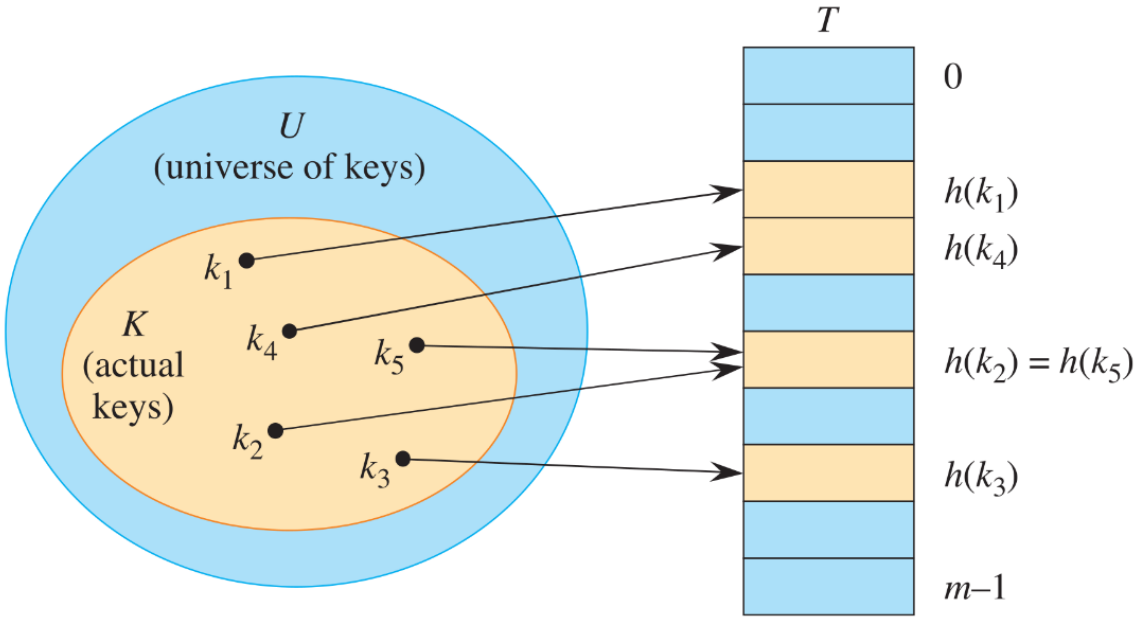

Chaining

Store all elements hashing to slot j in a linked list at $T[j]$.

```
T
0 --> NIL
1 --> [49] --> [86] --> NIL
2 --> [17] --> NIL
3 --> NIL
4 --> [52] --> [30] --> NIL
```

Chaining

```
CHAINED-HASH-INSERT(T, x)
    insert x at head of T[h(x.key)]

CHAINED-HASH-SEARCH(T, k)
    search list T[h(k)] for key k

CHAINED-HASH-DELETE(T, x)
    delete x from list T[h(x.key)]
```

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- Under SUHA, expected chain length at any slot $= \alpha$.

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Key Takeaway

If $n = O(m)$ (i.e., $\alpha = O(1)$), all operations take $O(1)$ expected time.