

Data Structures and Algorithms

Lecture 20

Aniket Basu Roy

2026-03-16 Mon

Contents

1	Agenda	2
1.1	Graphs	2
2	What is a Graph?	2
2.1	Examples	2
3	Basic Definitions	3
3.1	Adjacency	3
3.2	Degree	3
3.3	Paths and Cycles	3
3.4	Connectivity	3
3.5	Special Graphs	4

1 Agenda

1.1 Graphs

- Introduction and motivation
- Basic definitions and terminology

2 What is a Graph?

- A **graph** $G = (V, E)$ consists of:
 - V : a set of **vertices** (also called nodes)

- E : a set of **edges** (pairs of vertices)
- **Undirected graph**: edges are unordered pairs $\{u, v\}$
- **Directed graph** (digraph): edges are ordered pairs (u, v)
 - (u, v) means an edge from u to v ; u is the **tail**, v is the **head**

2.1 Examples

- Road networks: intersections = vertices, roads = edges
- Social networks: people = vertices, friendships = edges
- Web graph: pages = vertices, hyperlinks = directed edges
- Task scheduling: tasks = vertices, dependencies = directed edges

3 Basic Definitions

3.1 Adjacency

- In an undirected graph, u and v are **adjacent** if $\{u, v\} \in E$
- In a directed graph, vertex v is **adjacent from** u if $(u, v) \in E$

3.2 Degree

- **Degree** $\deg(v)$: number of edges incident to v
- In a directed graph:
 - **In-degree** $\deg^-(v)$: number of edges entering v
 - **Out-degree** $\deg^+(v)$: number of edges leaving v
- Handshaking lemma: $\sum_{v \in V} \deg(v) = 2|E|$

3.3 Paths and Cycles

- A **path** from u to v : sequence of vertices $u = v_0, v_1, \dots, v_k = v$ with each consecutive pair an edge
 - **Length** of path = number of edges k
 - **Simple path**: all vertices distinct
- A **cycle**: path where $v_0 = v_k$ and all other vertices distinct

3.4 Connectivity

- Undirected graph: **connected** if there is a path between every pair of vertices
- Directed graph:
 - **Strongly connected**: path from u to v and from v to u for all u, v
 - **Weakly connected**: connected if directions are ignored

3.5 Special Graphs

- **Tree**: connected undirected graph with no cycles; $|E| = |V| - 1$
- **Forest**: acyclic undirected graph (collection of trees)
- **DAG**: Directed Acyclic Graph — directed graph with no directed cycles
- **Bipartite graph**: $V = A \cup B$, all edges between A and B (none within)
- **Complete graph** K_n : every pair of vertices is connected; $|E| = \binom{n}{2}$